Natural convection in sidearms of lakes and reservoirs

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Motivation

Sidearms of Lakes and Reservoirs are important areas when dealing with water quality management. This research investigates sidearms through numerical modelling. In order to perform this modelling an Immersed Boundary Method has been implemented and tested.

The sidearm is simulated by a three dimensional domain with solar radiation represented by an internal heating term and a bottom boundary heat flux. Natural convection drives stong lateral flows mixing the sidearm with the main body of the lake.

Outline

- Introduction
- Numerical Method and Boundary Conditions
- Domain
- Results
- Future Work

Introduction

- Sidearms important due to inflow of nutrients and suspended particles
- Low thermal mass increases diurnal heating and cooling
- Solar radiation drives lateral circulation, mixing sidearm with main water body
- Sidearms typically under resolved in large lake numerical simulations

Numerical Method

Navier-Stokes equations

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

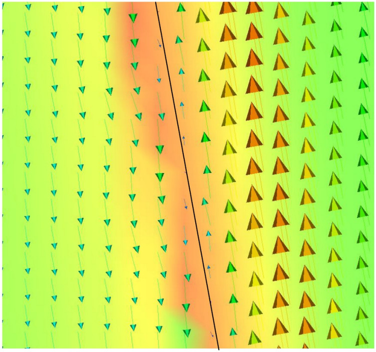
$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho_{ref}} \nabla P + \frac{\rho - \rho_{ref}}{\rho_{ref}} \mathbf{g} + \nu \nabla^2 \mathbf{u} + \mathbf{f_m}, \tag{2}$$

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \frac{\nu}{Pr} \nabla^2 T + f_e. \tag{3}$$

- Solved using finite volume, fractional step, pressure correction method
- 4th order Central Difference scheme for momentum
- 4th order Central Difference with ULTRA flux limiter for scalars
- 2nd order accurate in time

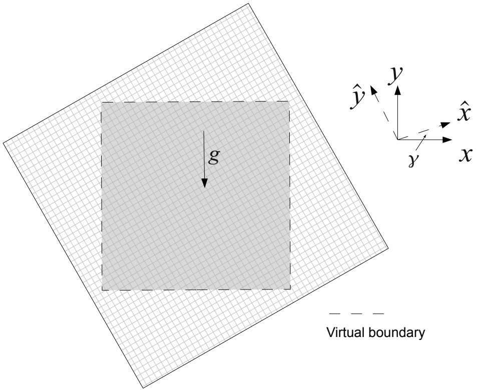
Immersed Boundary Method

- The IBM involves modifying the Navier Stokes equations
- A forcing function is used to create a virtual boundary
- The boundary does not necessarily have to coincide with grid points
- Much less computationally expensive than other similar methods
- Commonly used in simulations involving complex objects



Testing the IBM

- Square cavity used to validate an Immersed Boundary Method (IBM)
- Differentially heated side walls (Dirichlet boundary condition)
- Adiabatic top and bottom walls (constant flux (Neumann) boundary)



IBM Procedure

Calculate forcing term by comparing desired boundary value to actual boundary value

$$\mathbf{f}_{\mathbf{m}(i,j)} = \frac{1}{\delta t}\frac{1}{N_b}\sum_{n=1}^{N_b} D_{i,j}(\mathbf{x}_s)[\mathbf{V}-\mathbf{U}(\mathbf{x}_s)]+(\mathbf{u}\cdot\nabla\mathbf{u})_{i,j}+\nabla P_{i,j}-\nu(\nabla^2\mathbf{u})_{i,j} \quad (4)$$

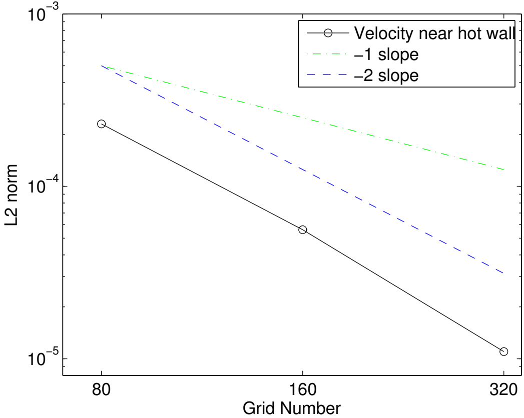
Dirichlet boundary condition

Temperature and velocity values are constant

Neumann boundary condition

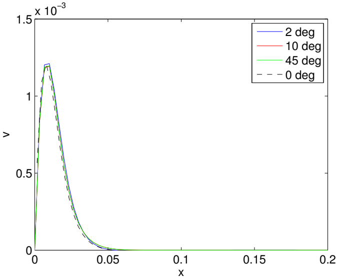
- An extra layer of virtual nodes is created
- Desired value is calculated at each step to give correct gradient

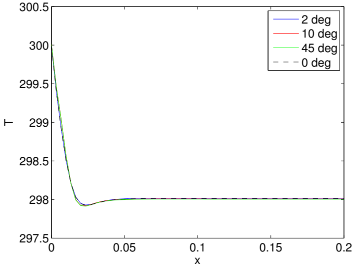
IBM Accuracy



- IBM 2nd order accurate
- L_2 norm based on vertical velocity near the hot wall versus grid resolution

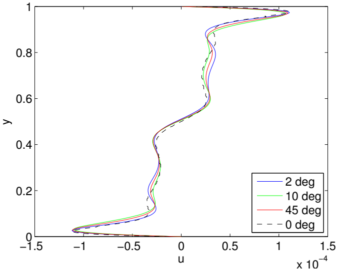
Hot wall v and T profiles

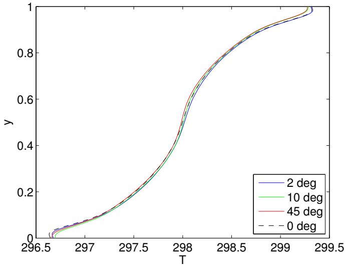




Velocity and Temperature at cavity mid-height

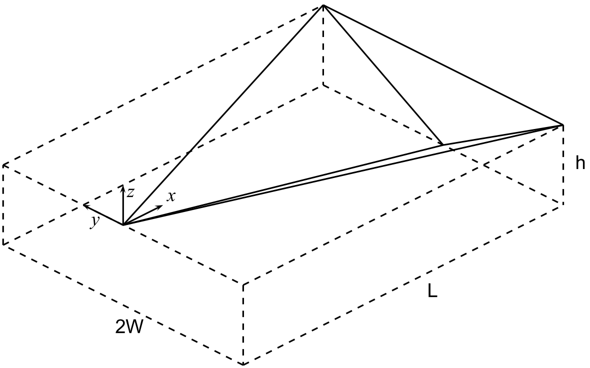
Cavity centre u and T profiles





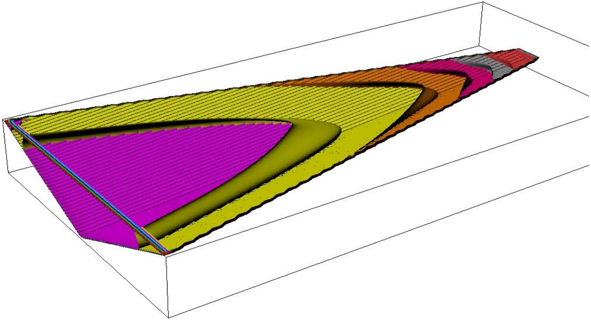
Velocity and Temperature at cavity centre

Sidearm Simulations



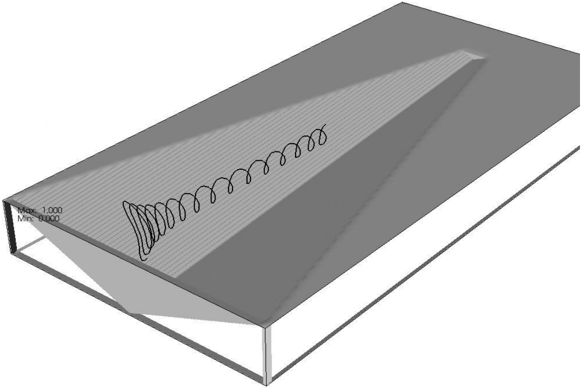
- Three-dimensional tetrahedron
- IBM used on bottom sloped surfaces
- Solar radiation model provides internal source term and bottom boundary heat flux

Sidearm Simulations



- Temperature contours
- Flow tends to move up the bottom surface and out along the top surface
- Warmest near the shallow tip due to low thermal mass

Sidearm Simulations



- Streamline starting near the shallow tip
- Domain shape causes complex flow structures

Conclusion

- Immersed Boundary successfully used to simulate a differentially heated cavity
- Simulations of a sidearm conducted using IBM for bottom boundaries
- Sidearm simulations generally show anticipated flow structure
- Additional complex flow structures also found

Future Work

- Further analysis of tetrahedron domain
- Extend simulations to include more complex, real world domains